Homework #1

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January 25, 2005

1. Find the polynomial of degree less than or equal to 2 which best approximates the function $f(x) = e^{-x}$ in the interval $[0, 1]$ in the L_2 sense.

2. Find the Fourier coefficients $\hat{u}_k$ of the function $u(x)$ defined by $u = x$ for $0 \leq x < \pi$,$u = x - 2\pi$ for $\pi \leq x \leq 2\pi$; check that $|k\hat{u}_k| \rightarrow$ a constant as $|k| \rightarrow \infty$.

3. Find the Fourier transform of the function $e^{-|x|}$.

4. Find the point in the plane $x + y + z = 1$ closest to (0,0,0). Note that this plane is not a linear space, and explain how our standard theorem applies.